HARPOON MISSILE AIRBORNE COMMAND AND LAUNCH SYSTEM AVAILABILITY MODEL.

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THESIS

HARPOON MISSILE AIRBORNE COMMAND AND LAUNCH SYSTEM AVAILABILITY MODEL

by

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Harpoon Missile Airborne Command and

Launch System Availability Model

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NOMENCLATURE

α	Maintenance efficiency
A	Availability
$\bar{A}$	Mean value of HACLS availabilities, A_i
A_{est}	Availability computed by the simulation
AAIU	Auxilliary Armament Interconnect Unit
ASW	Anti-Submarine Warfare
BIT	Built-in-Testing
DFR	Decreasing Failure Rate
F_d	Detected HACLS Failure
F_u	Undetected HACLS Failure
HACLS	Harpoon Airborne Command and Launch System
HDP	Harpoon Data Processor
HLU	Harpoon Logic Unit
HSCLS	Harpoon Shipboard Command and Launch System
HSTAT	HACLS Status
IFR	Increasing Failure Rate
IFRA	Increasing Failure Rate Average
I_i	Random Variable, Length of Idle Period
IMA	Intermediate Maintenance Activity
IX	A variable indicating simulation event code
k	Environmental Stress Factor
λ	Failure Rate
λ_L	Laboratory Failure Rate

λ^*	Critical Mission Failure Rate
LCB	Lower Confidence Bound
LCC	Life Cycle Costs
μ	Mean Time to Repair
MCP	Missile Control Panel
MDAC-E	McDonnell-Douglas Astronautics Company-East
MSTOP	A GASP variable used to stop the simulation
MTTF	Mean Time to Failure
MTTR	Mean Time to Repair
NALERT	A variable to count number of alert launches
NOK	A variable to count number of successful launches
O_i	Random variable, Length of Operating Period
OK	HACLS Up
RAM	Repairability, Availability, and Maintainability
SAR	Search and Rescue
T_i	Time to Fail in HACLS Mode, i
TTF	Time to Failure
TTR	Time to Repair
T_{up}	Total System Up Time
U	Random variable, Time to Repair
WRA	Weapon Replaceable Assembly

I. INTRODUCTION

Weapons availability and reliability studies are appearing frequently of late, much of the interest being motivated by the Air Force Project ABLE [1] which attempts to quantify the effects of reliability, availability, and maintainability (RAM). Project ABLE points out the need to improve RAM of new systems and to provide RAM growth for existing ones in order to reduce system life cycle costs (LCC), albeit at increased initial aquisition costs [2] .

In order to more accurately determine LCC, a reasonable effort to calculate weapon system reliability and availability is needed. For existing systems the calculation depends chiefly on gathering maintenance data, available in the Navy from its 3-M maintenance system data bank. For a new system, however, lack of such operational data requires that an analyst model the proposed system operation, make certain assumptions, and base calculations on reasoned judgment concerning projected system operation.

This paper outlines two models of Naval aircraft squadron operations from which single subsystem availability is determined for the Harpoon missile airborne command and launch system (HACLS). HACLS is a system composed of four weapon replaceable assemblies (WRAs), and various other components. The WRAs are missile control panel (MCP), Harpoon data processor (HDP), auxilliary armament interconnect unit (AAIU),

and Harpoon logic unit (HLU). HACLS has a built-in testing feature (BIT) which affords the operator a functional check of the system before and during a HACLS mission, while creating the most severe operating environment for the system.

As with all avionic systems HACLS is subject to periodic failures which render it unavailable for performing its designed functions. The resultant availability determined by the models is the probability that HACLS is operational on a randomly selected aircraft at a randomly selected time, given that the aircraft is functional. With suitable changes in mean time to failure (MTTF) and other critical input parameters the models can be exercised to give an availability figure for any aircraft subsystem.

The primary model, analytical in nature, is supported by a digital computer simulation model. After these models have been developed for the reader, parametric results are presented and an application of the resultant availability is shown.

The concept of availability is explained and its application to the models is shown. Although availability is developed in the classical manner, some different perspectives are developed. One of these is the notion of probability that HACLS is up at the beginning of a scheduled operating period. Others are operational availability and logistic availability.

Reliability data for the Harpon missile system, provided by McDonnell-Douglas Astronautics Company-East (MDAC-E), is used as an input to the availability models developed herein.

II. AVAILABILITY

A. GENERAL

To develop the concept of availability the following stochastic process is defined. Let $\{W(t), t \geq 0\}$ denote a performance process, such that

$$W(t) = 1 \text{ if the device is functioning at time, } t.$$
$$= 0 \text{ if the device is failed at time, } t.$$

Figure 1 shows this performance process.

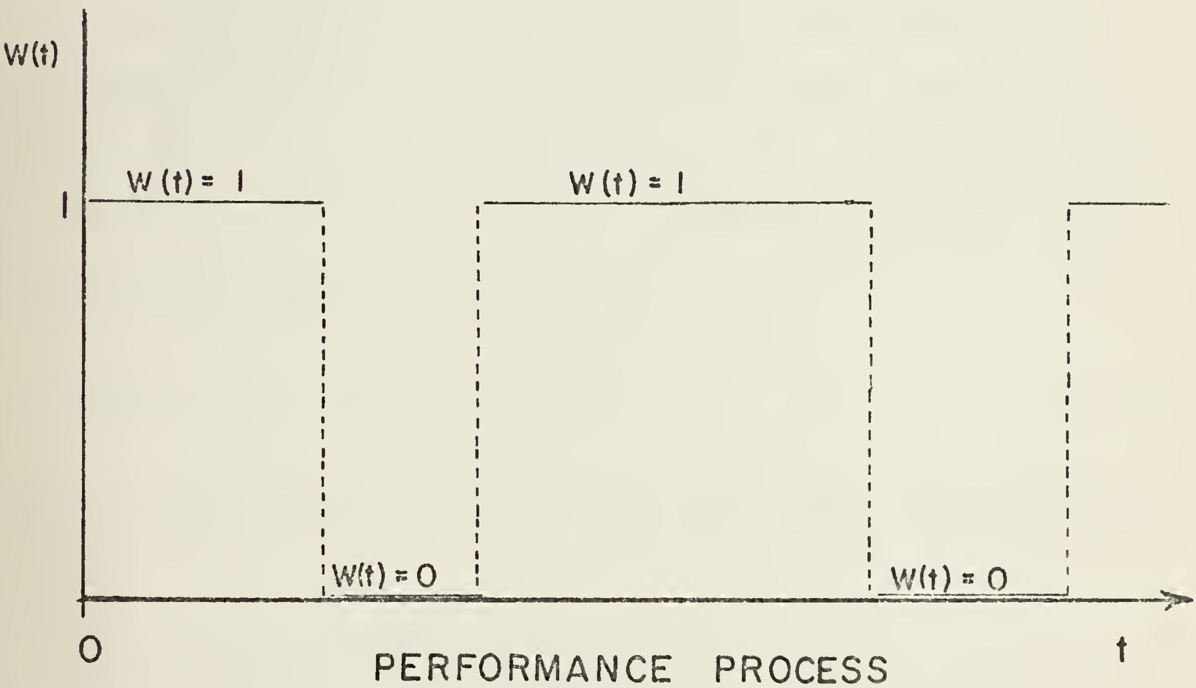


FIGURE 1

Let $A(t) = Pr[W(t) = 1]$, and $A(0) = 1$, i.e. $A(t)$ denotes the probability that the device is functional at time t. Note that $E[W(t)] = A(t)$.

Define

$$A = \lim_{t \to \infty} A(t)$$

if the limit exists. Therefore, A could be considered the probability that the device will function without regard to when its performance is required. This limiting availability, A, is in practice, most often called "the availability" but a more precise term for it is asymptotic availability.

In developing the models described in this paper an important variable is the proportion of time a device or system is in an up, i.e. functional, state. This proportion is derived as follows: if $T_{up}(t)$ represents the total up time of the system by time t, then

$$T_{up} = \int_0^t W(s)\, ds.$$

The expected proportion of up time from time = 0 to time = t is then

$$E[T_{up}(t)/t] = [\int_0^t A(s)\, ds]/t.$$

A theorem relating A and expected proportion of up time follows.

Theorem

Suppose $A = \lim_{t\to\infty} A(t)$ exists. Then,

$$\lim_{t\to\infty} T_{up}(t)/t \text{ exists,}$$

and is equal to A.

Proof of the above is theorem is given in Appendix A.

It can be shown [3] that with exponential times to failure (TTF) and exponential times to repair (TTR),

$$A(t) = \frac{1/\mu}{\lambda+1/\mu} + \frac{\lambda}{\lambda+1/\mu} \exp[-(\lambda + 1/\mu)t], \quad t \geq 0;$$

where

λ = failure rate = MTTF^{-1},

μ = mean time to repair, MTTR.

Thus,

$$A = \lim_{t\to\infty} A(t) = \frac{1/\mu}{\lambda+1/\mu},$$

and, after simplification, the final form of A is

$$A = \text{MTTF}/(\text{MTTF} + \text{MTTR}). \tag{1}$$

B. USE OF AVAILABILITY THEORY IN THE MODELS

Equation 1 holds for more general cases than exponential failure and repair times. In particular it holds when times to failure and times to repair are independent, identically distributed random variables.

In the first of the following models HACLS availability is defined as

$$A = Pr[\text{HACLS is up at a randomly selected time}].$$

This limiting probability is determined using a semi-Markov process.

The second model, a computer simulation, calculates availability in two ways:

(i) By using a mathematical expression equivalent to equation 1.

(ii) The status of HACLS systems is sampled at random times, thus creating a sequence of Bernoulli trials. After an adequate number of trials are performed, the number of successful trials is divided by the total number of trials, the result being an unbiased estimate of A. That is

$$A_{est} = N_{successes}/N_{trials}, \tag{2a}$$

or equivalently,

$$A_{est} = 1 - (N_{failures}/N_{trials}). \tag{2b}$$

Equation 2a provides availability by the following argument: Since for a Bernoulli random variable, W, with success probability, p, $E[W] = p$; the quotient in equation 2a is an unbiased point estimate of p. In an availability context p would be replaced by A(t) since, by definition, W(t) of the stochastic process $\{W(t), t \geq 0\}$ is a Bernoulli random variable when sampled at time t. Also, a sufficient number of trials is run, creating a limiting process which nullifies the time dependence of A(t), thus giving the desired result, a point estimate of A.

The expected range of availability values for HACLS is 0.95 - 0.999.... Therefore, an adequate number of simulated trials, assuming normal sampling theory is computed as follows.

If A_{est} is the point estimate of A determined by the simulation, and a nominal value of 0.975 is used for A, a lower confidence bound (LCB) may be computed. Recalling that A_{est} is an estimate of the Bernoulli parameter, A, the applicable 90% LCB is $A_{est} - \varepsilon$, where

$\varepsilon = Z_{0.90}[A(1-A)/n]^{1/2}$, $Z_{0.90}$ being the 90^{th} percentile of the standard normal distribution, implying a 90% LCB. Thus, with $\varepsilon = 0.005$, and after solving the above equation for the sample size, n,

$$n = (Z_{0.90}/\varepsilon)^2\ A(1 - A)$$

$$= (1.2815/0.005)^2\ (0.975)(0.025) = 1601.$$

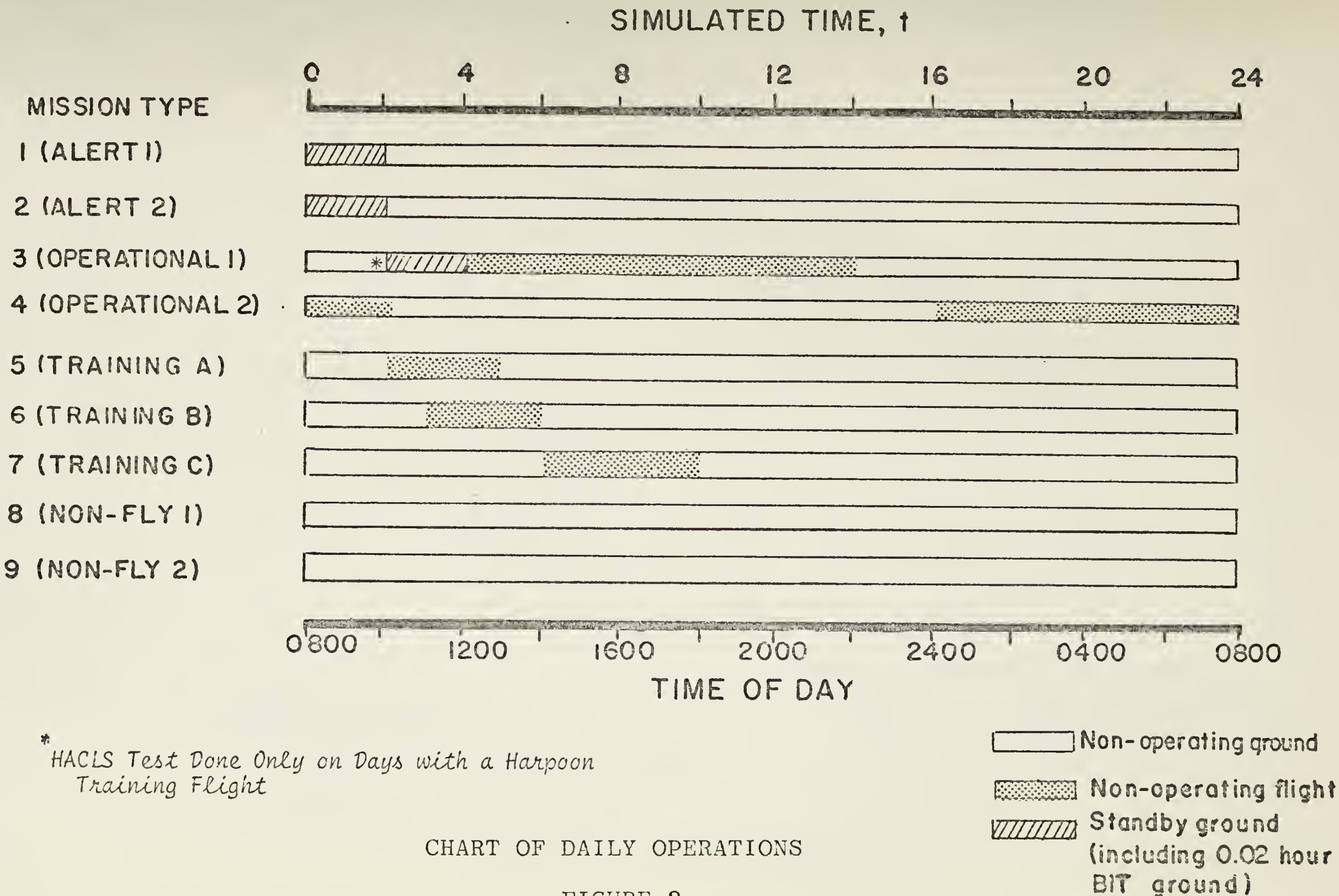
Therefore, the number of simulation runs is 1601. For the remainder of this discussion "A" will be used to denote both A_{est} and asymptotic availability, the use being made clear from the context.

III. FRAMEWORK OF SQUADRON OPERATIONS

Figure 2 shows the daily scenario of operations for a P-3 squadron. Time spent in each of the HACLS operating modes is shown for each aircraft. HACLS is checked for proper operation on both of the alert aircraft and on any aircraft that flies a Harpoon mission. These checks consist of a ground operating check, during which BIT is exercised, plus ground operation for the remaining preflight period. Should an aircraft fly with Harpoon, BIT is used one additional time prior to the missile launchings. All except two hours of the remaining flight time is spent in in-flight standby mode of HACLS operation. Note that when a HACLS flight is parametrically inserted into daily operations, a total of three preflight checks occur on that day. Moreover, the aircraft flying mission 3 has eight hours of HACLS inflight operation.

Figure 3 depicts the HACLS transition trees. It shows the transit from the three states: OK (HACLS up), F_u(undetected failure), and F_d(detected failure-in repair) to those same states on a subsequent day. Primes indicate transitional states.

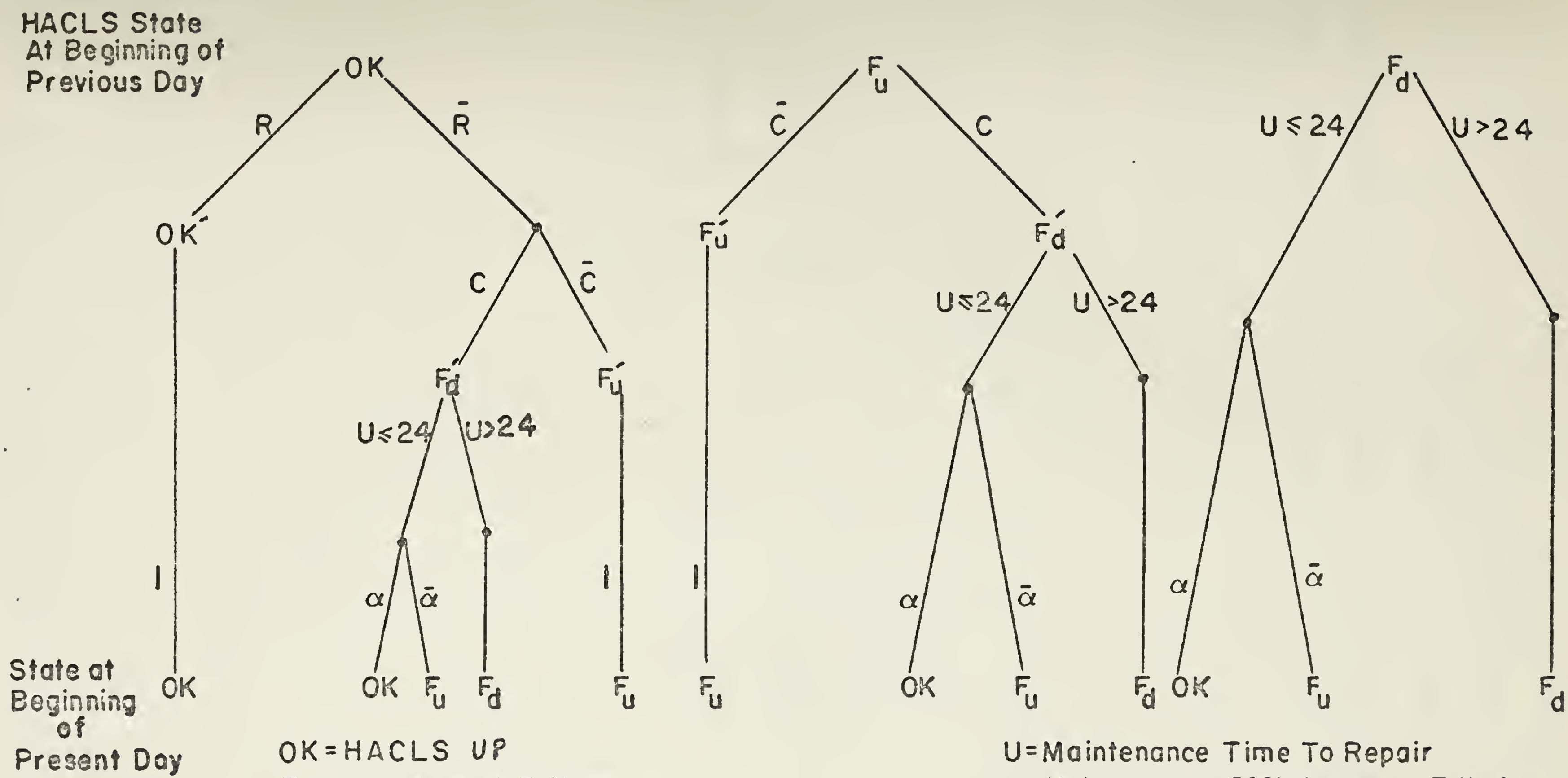
Considering that HACLS starts OK, and functions reliably, it transits to OK. If however it fails, it can go into F_d, the repair state, and if it is repaired before the next day,



* *HACLS Test Done Only on Days with a Harpoon Training Flight*

CHART OF DAILY OPERATIONS

FIGURE 2



OK = HACLS UP
F_u = Undetected Failure
F_d = Detected Failure
C = The HACLS was checked for proper operation
$\bar{C}$ = The Aircraft was not checked

U = Maintenance Time To Repair
α = Maintenance Efficiency; a Failed WRA is Actually Repaired
′ = Transitional States

R = Reliability

HACLS TRANSITION TREES

FIGURE 3

the system is again OK. If, however, maintenance action is completed but due to poor workmanship, the system is not in fact repaired, it transits to F_u. On the other hand, if maintenance is not completed by the beginning of the present day, HACLS remains F_d. Finally, if HACLS is not preflight checked, the system remains F_u. The remaining trees of figure 3 show the transits from F_u and F_d to the possible states of the present day.

In terms of Markovian theory, the transitions depicted form a positive recurrent, aperiodic, and irreducible Markov process, i.e. it is irreducible ergodic. This property becomes cruicial in the development of the following analytical model. Note that the difference between F_u and F_d depends on whether the HACLS is checked, i.e. operated.

IV. ANALYTICAL MODEL

A model developed by MDAC-E determines the availability of the Harpoon shipboard command and launch system (HSCLS)[4]. The system, as envisioned, is to be checked daily for proper operation, and furthermore, it may be subjected to use on an alert mission. This model is developed in a straight-forward way since the time of shipboard daily operational system test periods is somewhat predictable. In developing a model for an aircraft squadron, the problem changes, since there are nine or more aircraft, whose HACLS are checked infrequently and in a non-deterministic fashion. Therefore, the following model is developed.

This section describes an analytical model of the operations of three types of Navy aircraft squadrons. The objective of the model is to determine the availability of the Harpoon missile airborne command and launch system (HACLS). This model gives availability of the HACLS subsystem given that its parent system, i.e. the aircraft, is functioning properly. Availability in this context is defined as the probability that the HACLS is in functional condition at a randomly selected point in time.

A. ASSUMPTIONS

As formulated this model is responsive to:

1) HACLS Reliability (R). In accordance with accepted reliability theory it is assumed that the exponential

distribution adequately describes the time to failure of most avionic systems. Furthermore, the exponential distribution readily lends itself to analytical modelling. Therefore, reliability would be of the form,

$$R = \Pr[T_i > t_i] = \exp(-k_i \lambda_L t_i),\ t_i \geq 0.$$

The random variable T_i, HACLS time to fail when in operating mode i, is assumed to be exponential and the system failure rate is $k_i \lambda_L$. In general a failure rate may be a function of time, i.e. $\lambda = \lambda(t)$. This dependence on time gives rise to decreasing failure rate (DFR) distributions, increasing failure rate (IFR) distributions, and others. In this model, however, λ_L, the laboratory failure rate is assumed to a constant equal to 0.0011 [failures per hour] over all HACLS modes of operation, but the multiplier k_i varies according to the severity of the operational environment. The vector of environmental k-factors is shown as Table I.

A high k-factor indicates a more severe mode of operation and vice versa. A scalar k-factor, k_{avg}, based on a time - weighted average of the k_i's, is used for the analytical model. In order to apply this k_{avg} one must assume that there exists no shock effect when HACLS goes from one operating mode to another. An additional assumption is that the derating effects of one mode of operation do not interact with those of another. The calculation of k_{avg} is included in Appendix B.

2) Maintenance Time to Repair (U). Although times to failure of avionic systems are generally assumed to be exponential, there exists some controversy over the distribution of U. It is popular among reliability analysts to assume that U has a log-normal distribution. However, a log-normal distribution presents mathematical difficulties, causing the resulting model to become analytically intractable. Moreover, as Thompson points out in [5], very often samples from log-normal and exponential distributions are indistinguishable. Finally, if in fact the distribution of U is log-normal, the assumption of an exponential U with a constant MTTR, μ, would yield a valid point estimate of availability by the following argument. It is clear from equation 1 that asymptotic availability depends not on the distributions of TTF and TTR, but on their means. Therefore, if one accepts equation 1 with its aforementioned

TABLE I

A/C TYPE	HACLS OPERATING MODE	NON-OPERATING STORAGE	FLIGHT	STBY GROUND	BIT GROUND	STBY FLIGHT	BIT FLIGHT
	MODE INDEX	1	2	3	4	5	6
P-3	k	0.01	0.1	1.0	6.0	5.0	6.0
S-3A	k	0.01	0.167	1.0	6.0	5.0	6.0
A-6	k	0.01	0.5	1.0	6.0	5.0	6.0

ENVIRONMENTAL - k FACTORS

restrictions, asymptotic availability, A, would be equal for the exponential TTF--exponential TTR and the exponential --- log-normal cases. Thus, in the analytical model U is assumed to be exponentially distributed with a mean time to repair, μ, equal to 24 hours.

3) Maintenance Efficiency (α). α is defined as the probability that HACLS is in fact repaired, given that repair action on the faulty WRA is completed. Equation 1 may be modified to account for maintenance efficiency;

$$A = MTTF / [MTTF + (MTTR/\alpha)], \tag{3}$$

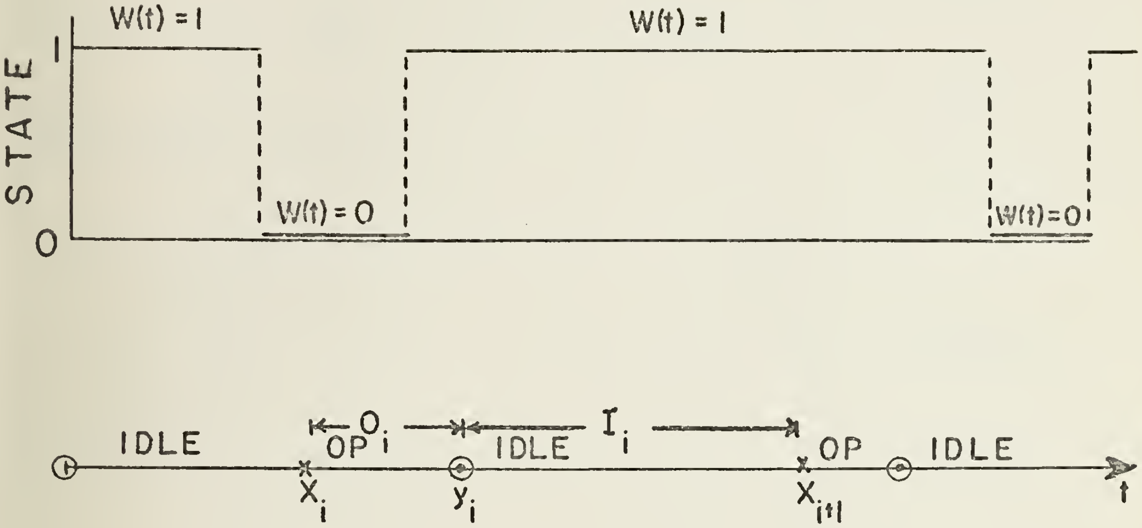
It is evident from equation 3, that as α increases, the asymptotic availability increases, an intuitively appealing result. The expected range of values for α is 0.80-0.95. To provide a worst-case analysis an α of 0.80 is chosen.

4) The HACLS operating scheme; i.e. the length of HACLS idle periods and operating period length. The operating periods are two hours or eight hours long for preflight checks and Harpoon flights, respectively. By specifying the number and frequency a probability mass function for length of operating periods may be deduced. The density function for idle period length may be determined through computer simulation.

The main question addressed by this paper is: How do parametric variations in length of idle and/or operating periods affect availability? The model developed to answer this question is a semi-Markov process, that is an alternating renewal process containing an imbedded Markov chain.

An example motivated by Cox and Miller [6] follows.

Suppose a piece of equipment can be in one of two states, 1 if it is up and 0 if it is down and in repair. It is assumed that the equipment is constantly monitored and that as soon as it fails it goes into repair (state 0). Figure 4 depicts a semi-Markov process representing the status of the equipment over time.



Simple Alternating Renewal Process With an Imbedded Markov Chain

FIGURE 4

The associated transition matrices, one for the X_i renewals and the other for the Y_i renewals, are as follows:

For X_i to Y_i,

$$Q = \begin{vmatrix} q_1 & 1 - q_1 \\ & \\ & \\ 1 - q_2 & q_2 \end{vmatrix}, \quad \begin{array}{l} 0 \leqslant q_1 \leqslant 1 \\ \\ 0 \leqslant q_2 \leqslant 1 \end{array}$$

and for Y_i to X_{i+1}

$$R = \begin{vmatrix} r_1 & 1 - r_1 \\ & \\ & \\ 1 - r_2 & r_2 \end{vmatrix}, \quad \begin{array}{l} 0 \leqslant r_1 \leqslant 1 \\ \\ 0 \leqslant r_2 \leqslant 1 \end{array}$$

Then for X_i to X_{i+1},

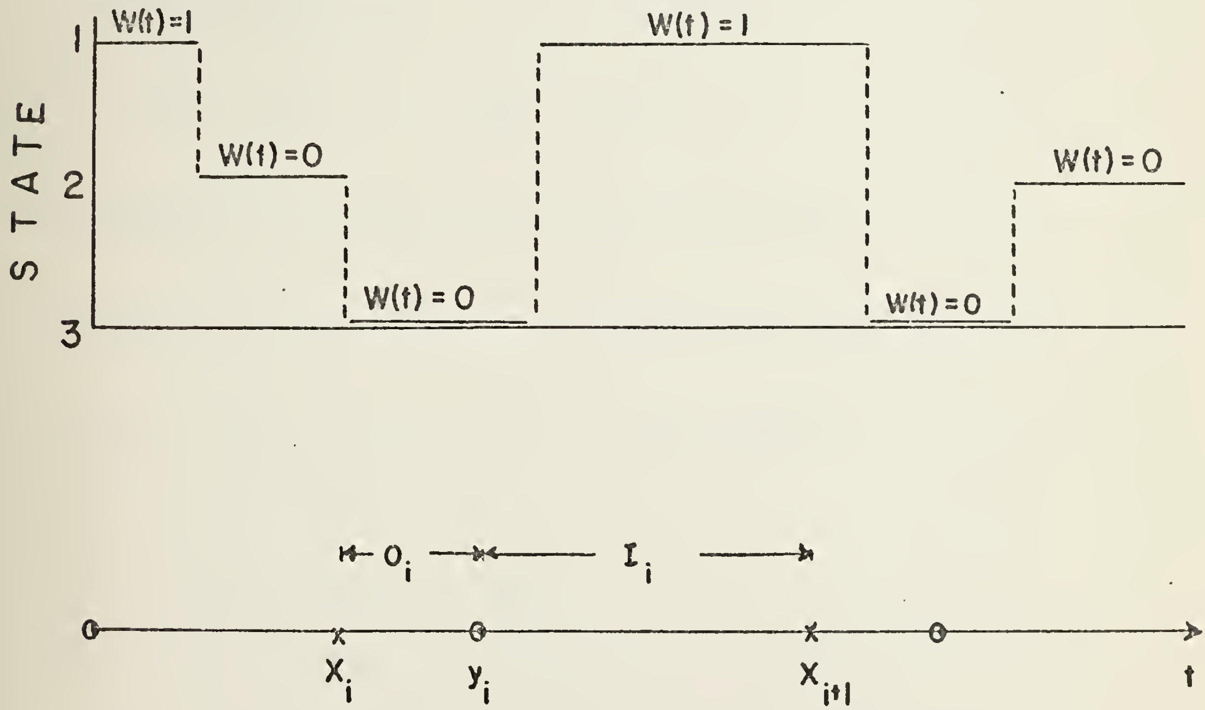
$$S = QR = \begin{vmatrix} q_1 r_1 + (1-q_1)(1-r_2) & q_1(1-r_1) + (1-q_1) r_2 \\ & \\ (1-q_2) r_1 + q_2(1-r_2) & (1-q_2)(1-r_1) + q_2 r_2 \end{vmatrix},$$

is the transition matrix.

B. MODEL DESCRIPTION

The model described in this paper considers one aircraft and tracks its HACLS status through time. The HACLS installed aboard that aircraft may be in any one of three states: 1. the functional state, 2. the failed state, and 3. the system-under-repair state. Furthermore, the system may be functioning or it may be idle, with some probability, at a given point in time. The functioning or operating periods are fixed both for preflight checks of HACLS and for Harpoon carrying missions. HACLS idle periods within the daily operations are distributed exponentially.

Figure 5 depicts the alternating renewal process used to model the status of HACLS.



Semi-Markov Process Used in the Analytical Model
FIGURE 5

The distinction between state 2 and state 3 is as follows: HACLS is subject to unobserved and observed failures. For the sample replication shown in Figure 5, if the system fails during an idle period, the system goes from state 1 to state 2 (undetected failure), but if the system fails during an operating period, it goes from state 1 to state 3, since the failure is immediately detected. Further, if the system were in state 2, it could transit to state 3 (when the system is exercised, and the failure discovered) and subsequently from state 3 to state 1.

To find availability, let $P_j(t)$ be the probability that the HACLS is in state j at time t. Then,

$$A(t) = P_1(t), \text{ and}$$

$$A = P_1, \text{ where } P_1 = \lim_{t\to\infty} P_1(t).$$

To determine P_1, define the state variables,

X_i = state at start of i^{th} operating period, and

Y_i = state at start of i^{th} idle period:

= 1 if HACLS is up;

= 2 if HACLS is down;

= 3 if HACLS is in repair.

Referring to figure 5, the transitions from X_i to Y_i are shown below.

If $X_i = 1$,

$Y_i = 1$ if HACLS is up at the end of O_i;

$= 2$ with probability zero;

$= 3$ if in repair at the end of O_i.

If $X_i = 2$ or $X_i = 3$,

$Y_i = 1$ if HACLS is up at the end of O_i;

$= 2$ with probability zero;

$= 3$ if in repair at the end of O_i.

A description of the transitions from Y_i to X_{i+1} follows.

If $Y_i = 1$,

$X_{i+1} = 1$ if HACLS remains up through I_i;

$= 2$ if it fails in I_i;

$= 3$ with probability zero.

$Y_i = 2$ with probability zero.

If $Y_i = 3$,

$X_{i+1} = 1$ if repair is successfully completed in I_i and the system stays up to the end of I_i;

$= 2$ if,

(i) repair is unsuccessfully completed in I_i, or

(ii) repair is successfully completed in I_i, but HACLS fails again before the end of I_i;

$= 3$ if repair is not completed in I_i.

The transitions from the X_i to the X_{i+1} renewal are described as follows.

If $X_i = 1$,

$X_{i+1} = 1$ if,

(i) HACLS stays up through O_i and I_i, or

(ii) HACLS fails in O_i, and is successfully repaired before the end of I_i;

$= 2$ if HACLS stays up through O_i, and fails in I_i;

$= 3$ if HACLS fails in O_i and repair is not complete by the end of I_i.

If $X_i = 2$,

$X_{i+1} = 1$ if HACLS is repaired by the end of I_i;

$= 2$ with probability zero;

$= 3$ if HACLS is not repaired by the end of I_i.

If $X_i = 3$,

$X_{i+1} = 1$ if HACLS is repaired by the end of I_i;

$= 2$ if HACLS is not successfully repaired by the end of I_i;

$= 3$ if repair action is not complete by the end of I_i.

Define the transition matrices P_A, P_B, and P as follows:

P_A depicts the transition probabilities for X_i to Y_i,

P_B depicts the transition probabilities for Y_i to X_{i+1}.

and $P = P_A P_B$ depicts the transition from X_i to X_{i+1}. The

development of the entries of P_A, P_B, and P is included as Appendix C.

Letting $\lambda = k_{avg}\ \lambda_L$;

$\mathcal{O}(s)$ and $\mathcal{I}(s)$ be the Laplace transforms of operating and idle time probability mass and density functions, respectively; and further,

$$a = \frac{\lambda}{\lambda+1/\mu} + \frac{1/\mu}{\lambda+1/\mu}\ \mathcal{O}(\lambda+1/\mu),$$

$$b = \frac{1/\mu}{\lambda+1/\mu} + \frac{\lambda}{\lambda+1/\mu}\ \mathcal{O}(\lambda+1/\mu),$$

$$c = \frac{\alpha/\mu}{1/\mu-\lambda}\ [\mathcal{I}(\lambda) - \mathcal{I}(1/\mu)]\ ,$$

$$d = 1 - c - \mathcal{I}(1/\mu),$$

x = arbitrary non-negative entries;

$$P_A = \begin{vmatrix} a & 0 & 1-a \\ 1-b & 0 & b \\ 1-b & 0 & b \end{vmatrix}\ ,$$

$$P_B = \begin{vmatrix} \mathcal{I}(\lambda) & 1-\mathcal{I}(\lambda) & 0 \\ x & x & x \\ c & d & \mathcal{I}(1/\mu) \end{vmatrix}\ , \text{ and}$$

$$P=P_AP_B=\left|\begin{array}{ccc} aI(\lambda)+(1-a)c & a[1-I(\lambda)]+(1-a)d & (1-a)I(1/\mu) \\ (1-b)I(\lambda)+bc & (1-b)[1-I(\lambda)]+bd & bI(1/\mu) \\ (1-b)I(\lambda)+bc & (1-b)[1-I(\lambda)]+bd & bI(1/\mu) \end{array}\right| .$$

Since the Markov chain at points X_i is irreducible ergodic, there exists a solution to the following equation:

$$\pi = \pi(P_AP_B).$$

The elements of the vector, π, are as follows,

π_1 = Pr[OK at the start of operating period];

π_2 = Pr[F_u at the start of operating period];

π_3 = Pr[F_d at the start of operating period].

The solution for π_1 is

$$\pi_1 = \frac{(1-b)I(\lambda)+bc}{1-O(\lambda+1/\mu)[I(\lambda)-c]}$$

Solutions for π_2 and π_3, although readily calculable, can each be taken to be approximately equal to $(1-\pi_1)/2$.

To compute P_1 note that

π_j = the (long-run) proportion of transitions which are into state j, and define

w_j = the expected time spent in state j per transition.

Ross [7] develops an equation which yields P_1, the desired limiting probability,

$$P_1 = \pi_1 w_1 / \sum_{j=1}^{3} \pi_j w_j .$$

Rigorous development of the w_j's is beyond the scope of this paper, but if one assumes that $w_j = \pi_j \, E[O_i + I_i]$, for $j = 1, 2, 3$, a vector, w, may be calculated. For example with $E[O_i] = 2$ hours (no HACLS flights) and $E[I_i] = 90$ hours, $E[O_i + I_i] = 92$ hours. With

$\pi' = (0.949, 0.02505, 0.02505)$, which gives

$w' = (87.308, 2.346, 2.346)$, the resultant P_1 is 0.998.

V. COMPUTER SIMULATION MODEL

The computer simulation model is written in the GASP IV programming language [8]. It is a Monte Carlo simulation of a Naval P-3 aircraft squadron on an overseas deployment, engaged in peacetime operations. At randomly selected times of day, 1601 successive Harpoon alert flights are "launched" and the number of successful launches is divided by the number of attempted launches. The resulting quotient yields the availability, A.

The following is a description of simulated P-3 squadron operations, but suitable programming changes can be made to allow the user to apply it to S-3A or A-6 squadrons, as well. Availabilities discussed in this paper for the latter two squadrons are determined from the foregoing analytical model, however.

A. ASSUMPTIONS

1) This model assigns aircraft by identification number (1-9) to the various missions outlined in figure 2. All of these aircraft/mission assignments are shuffled at the completion of each simulated day, with the exception that one of the aircraft in the non-flying, grounded status remains so for one simulated week to satisfy a real-world aircraft (not HACLS) maintenance constraint.

2) The simulation causes alerts to be scheduled as follows: one per day at a random time between $t = 0$ and $t = 24$ hours. The strike force consists of the ready-alert 1 and 2 aircraft, and either the operational aircraft 1 or 2 depending on which one is not flying at the time of the alert.

3) Should the alert be scheduled during the first hour of the day, the aircraft from the respective missions of the previous day are assigned to the strike force.

4) One test bench is available for the repair of HACLS failed components. If the test bench is occupied, failed WRAs go into a maintenance queue.

5) Replacement spares are unavailable, and thus each failed WRA goes into an under - repair state as soon as failure is discovered.

6) GASP provides a set of parametrically controlled random variate generators giving the user a choice concerning the maintenance completion time probability distribution. For this simulation the maintenance completion time function is parametrically varied from an exponential to log-normal probability density, each with an arbitrary mean of 24 hours and standard deviation of 4 hours, since no empirical maintenance information yet exists on HACLS at the time of this writing.

7) As in the analytical model, time to failure, TTF, is assumed to be exponential, a distribution characterized by a constant failure rate for all values of time. The simulation program could be modified to provide for wear-in, decreasing failure rate (DFR), or wear-out, increasing failure rate (IFR), of HACLS. However, that modification is not made because it is unclear whether the system would consistently exhibit wear-in or wear-out throughout its lifetime, since the true distribution is probably of the increasing failure rate average (IFRA) class. The IFRA character is implied since HACLS is composed of elements which themselves probably have exponential TTFs [3]. An IFR distribution would imply wear-out, only. Thus, the no-wear exponential is used in order to avoid making a significant error in TTF determination.

B. CONCEPT OF GASP IV

References 8 and 9 provide examples of simulations and a full explanation of the GASP IV simulation language. The GASP language provides the user a tool with which he may model discrete, continuous, or discrete - continuous processes. Since the HACLS availability model is perceived to be discrete,

that mode of GASP is employed. A functional flowchart of GASP is shown as figure 6.

Figure 6 describes the logic flow of the GASP IV computer simulation language as it applies to a discrete event, variable time increment model. The user-supplied MAIN program contains a FORTRAN command, "CALL GASP," which it executes after each parametric change in the value of maintenance efficiency, α, is made. After GASP is called for each of the three values of α run, the following chain of events is set into motion:

1) Input cards are read, GASP initializes files and GASP variable arrays.

2) GASP calls subroutine INTLC to intialize non-GASP variables.

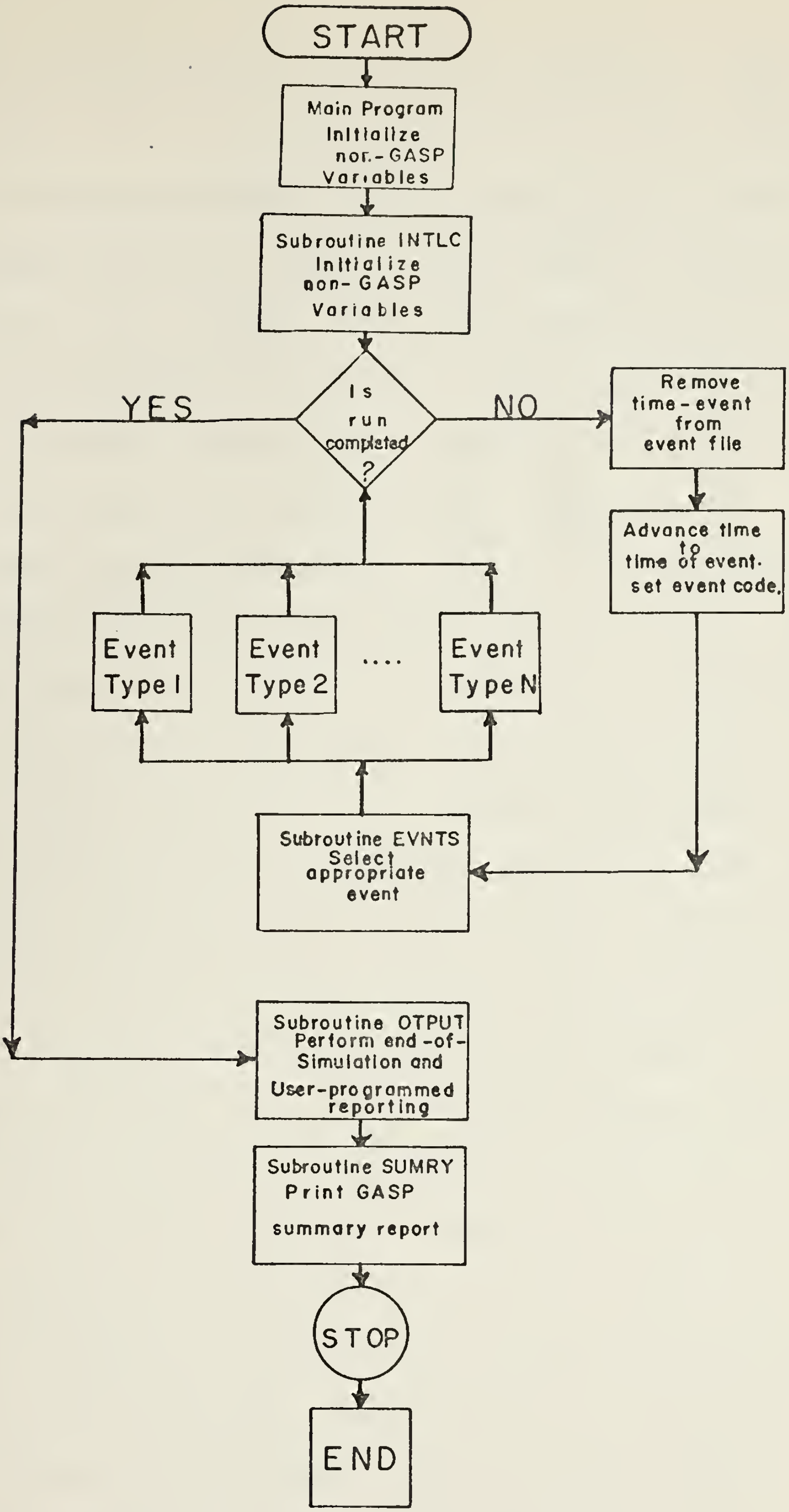
3) The simulation begins and GASP advances the clock to the time of the next event and generates the event code, IX.

4) GASP calls subroutine EVNTS(IX) and a computed GO TO statement calls the user-supplied subroutine corresponding to the event of code, IX.

5) GASP continues to advance the clock and schedule events as above, checking to determine if the stopping variable, MSTOP, has been set to a negative value. The program sets MSTOP equal to -1, after the desired number of replications, causing GASP to stop the simulation and print out the results of the run.

C. MODEL DESCRIPTION

The following is a description of this GASP IV simulation model. As before, the model utilizes an exponential reliability function, R_i, of $\exp(-k_j \lambda_L t_i)$, $i=1,...,9$, $t_{ij} \geq 0$.



Functional Flow Chart of GASP IV

FIGURE 6

Again the k-factors used are listed in Table I. Time, t_{ij}, is determined from the matrix shown in Table II. This T matrix represents the time during a simulated day that an individual aircraft, i(i=1,...,9), spends in operating mode, j(j=1,...,6). Also, there are assumed to be no start-up shocks when HACLS goes from one operating mode to another. Finally, as before, the derating effects of one mode do not interact with those of a mode entered subsequently.

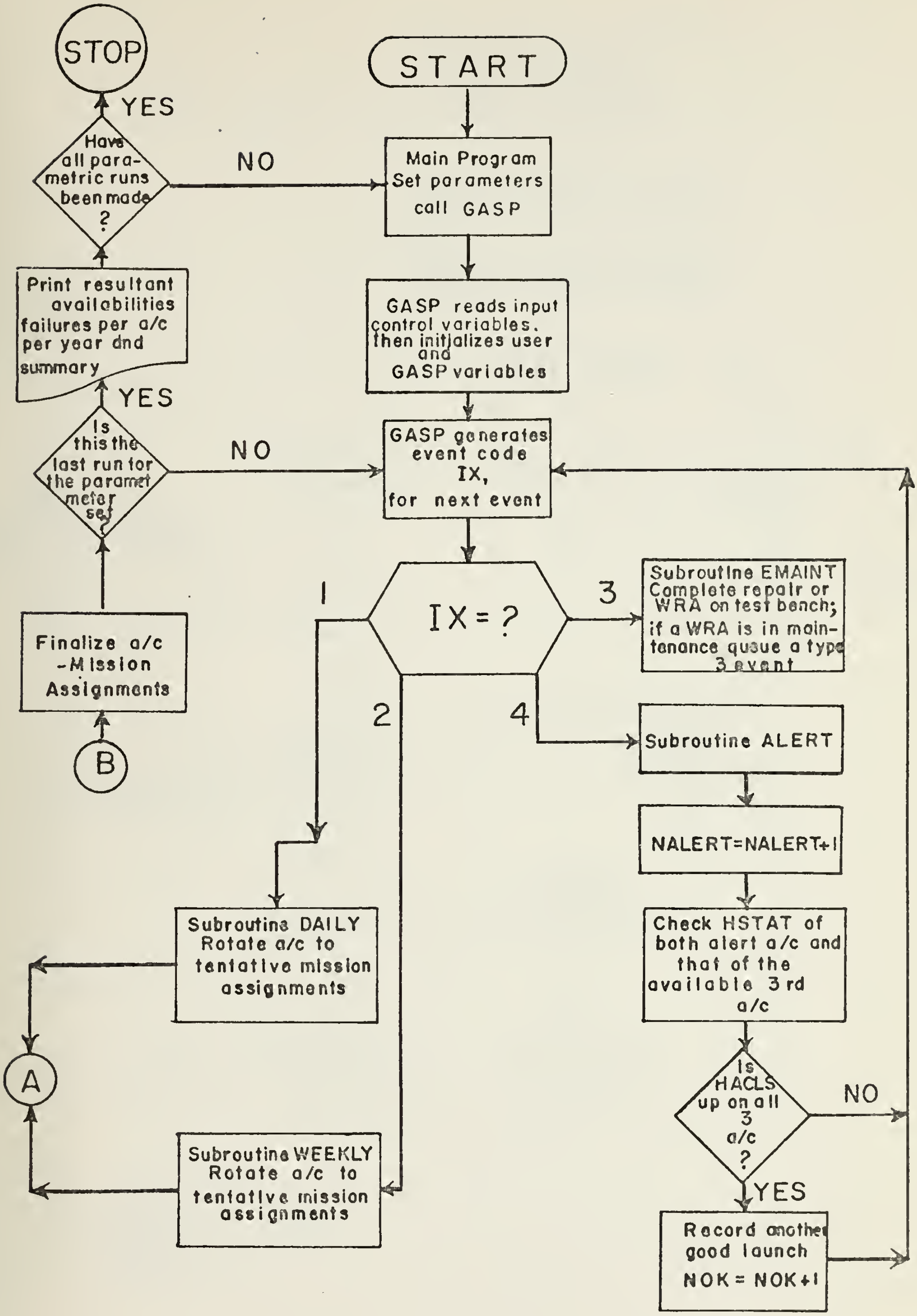
The model is run with daily rotation, weekly rotation, end of HACLS maintenance, and alert launch events. Figures 7 and 8 show the logic flow of the simulation. All aircraft start with UP HACLS. An initial DAILY event (IX=1) is forced in order to start the simulation. Subroutine STATUS is then called from subroutine DAILY to check for failures on each aircraft during the preceeding 24 hour operating period. If the HACLS has failed on an aircraft and its failure is discovered, subroutine MAINT(INDEX), where INDEX = aircraft side number, is called and an end of maintenance (subroutine EMAINT) event is scheduled. Moreover, if the failure occurred on an alert aircraft, subroutine REPLAC is called and another aircraft (HACLS UP) is substituted for it. After all aircraft have been checked for failures, the aircraft assignments of the new day are finalized. Upon return to subroutine DAILY, an alert launch is scheduled. On the basis of ZHOUR, the time of alert launch, a check of HACLS status (HSTAT) is made and the availability calculation is initiated. The launches are aggregated into either launch time less than

TABLE II

PREVIOUS DAY MISSION ASSIGNMENTS	HACLS MODE INDEX 1	2	3	4	5	6
1 (ALERT 1)	22	0	1.98	0.02	0	0
2 (ALERT 2)	22	0	1.98	0.02	0	0
3 (OPERATIONAL 1)	14	10	0	0	0	0
4 (OPERATIONAL 2)	14	10	0	0	0	0
5 (TRAINING A)	21	3	0	0	0	0
6 (TRAINING B)	21	3	0	0	0	0
7 (TRAINING C)	20	4	0	0	0	0
8 (NON-FLY 1)	24	0	0	0	0	0
9 (NON-FLY 2)	24	0	0	0	0	0

T MATRIX

Entries are the number of hours, by mission category, that HACLS is in the various operating modes.



Simulation Flow Chart for the Calculation of A_{est}

FIGURE 7A

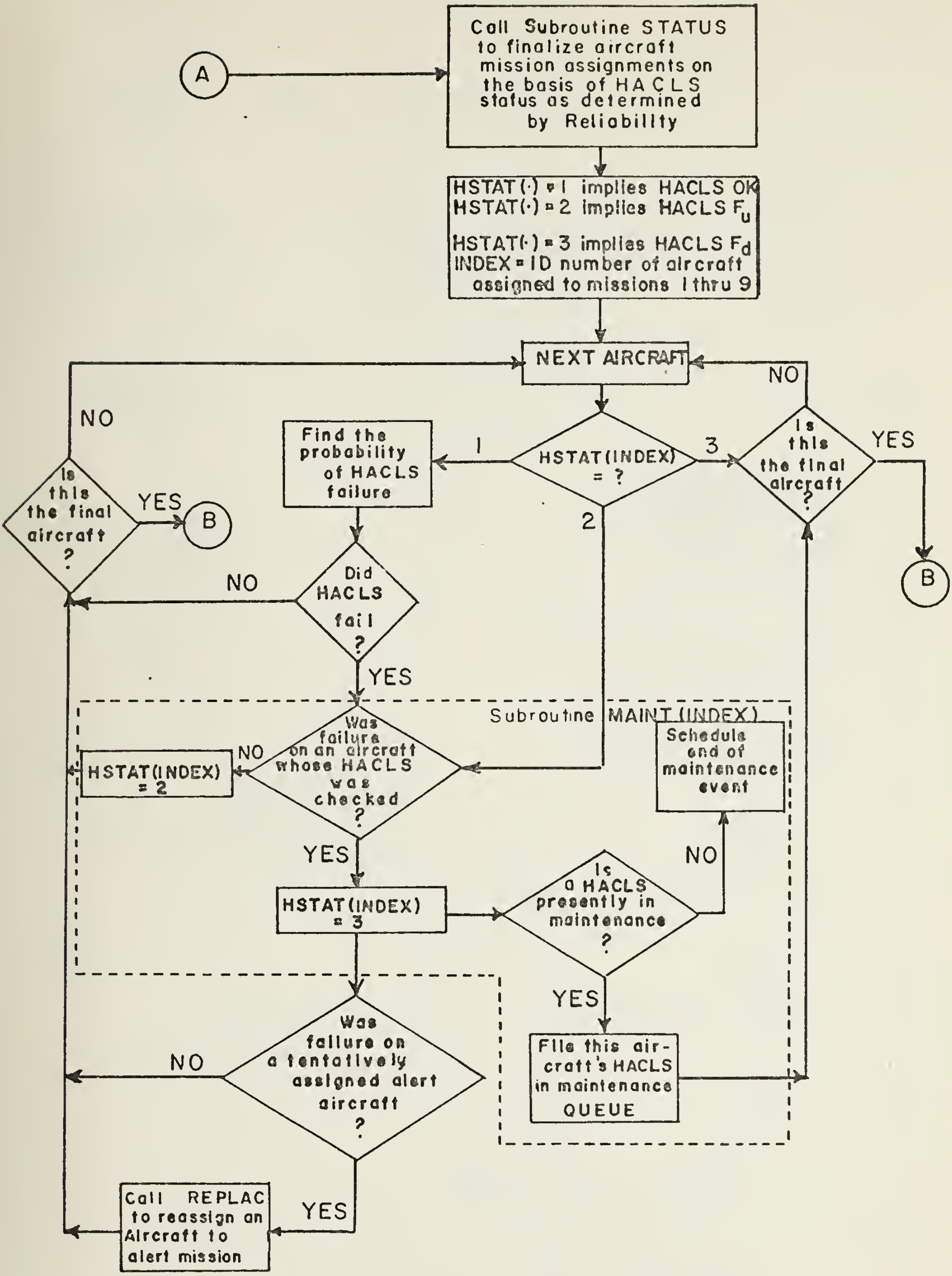
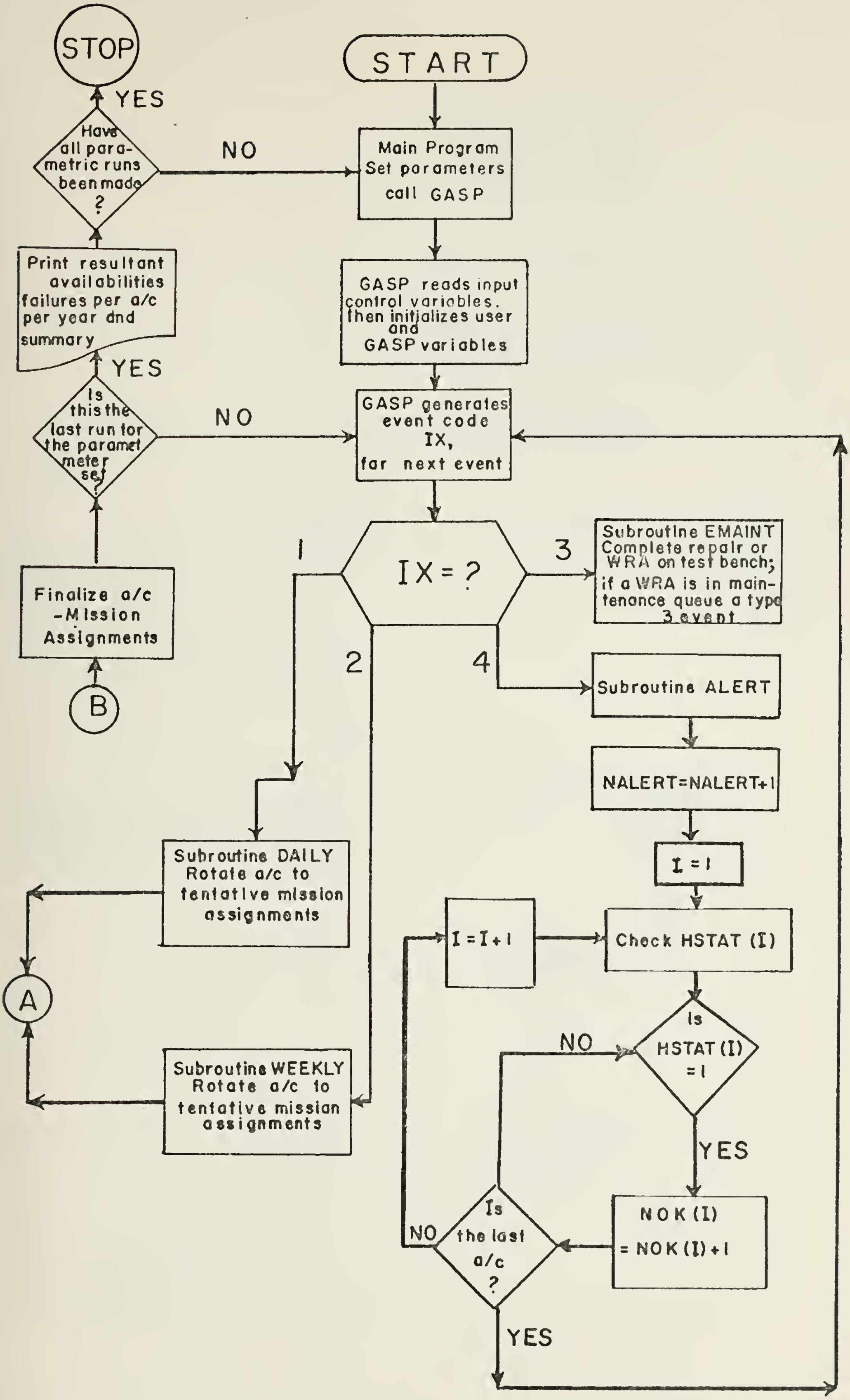

A
Call Subroutine STATUS to finalize aircraft mission assignments on the basis of HACLS status as determined by Reliability
HSTAT(·) = 1 implies HACLS OK
HSTAT(·) = 2 implies HACLS F_u
HSTAT(·) = 3 implies HACLS F_d
INDEX = ID number of aircraft assigned to missions 1 thru 9
NEXT AIRCRAFT
NO
HSTAT(INDEX) = ?
1
2
3
Find the probability of HACLS failure
Is this the final aircraft ?
YES
B
NO
Is this the final aircraft ?
YES
B
Did HACLS fail ?
NO
YES
Subroutine MAINT (INDEX)
Was failure on an aircraft whose HACLS was checked ?
NO
HSTAT(INDEX) = 2
YES
HSTAT(INDEX) = 3
Schedule end of maintenance event
Is a HACLS presently in maintenance ?
NO
YES
File this aircraft's HACLS in maintenance QUEUE
Was failure on a tentatively assigned alert aircraft ?
NO
YES
Call REPLAC to reassign an Aircraft to alert mission

FIGURE 7B



Simulation Flow Chart for the Calculation of $\bar{A}$

FIGURE 8A

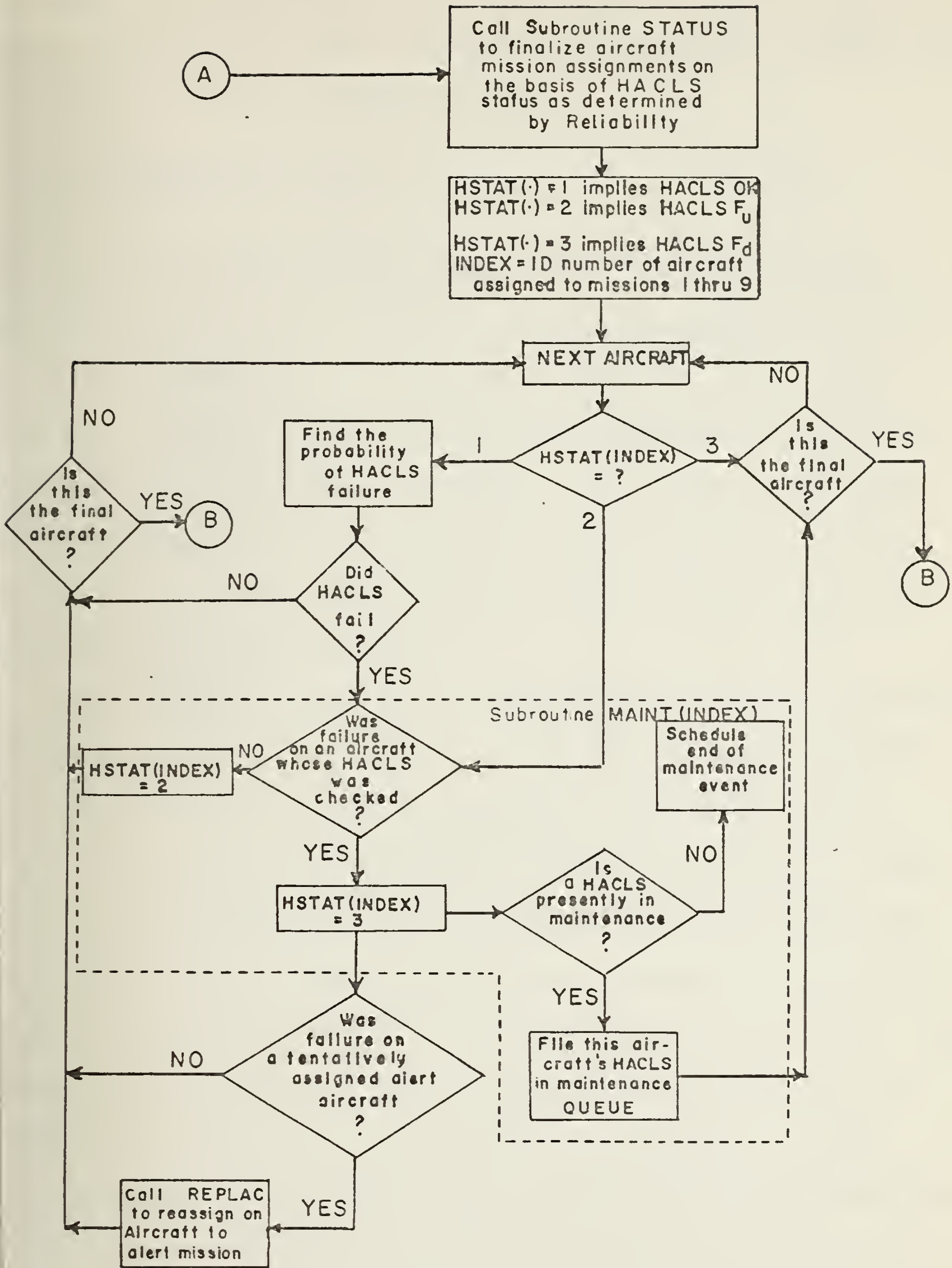

A
Call Subroutine STATUS to finalize aircraft mission assignments on the basis of HACLS status as determined by Reliability
HSTAT(·) = 1 implies HACLS OK
HSTAT(·) = 2 implies HACLS F_u
HSTAT(·) = 3 implies HACLS F_d
INDEX = ID number of aircraft assigned to missions 1 thru 9
NEXT AIRCRAFT
HSTAT(INDEX) = ?
1
2
3
Is this the final aircraft ?
YES
NO
B
Find the probability of HACLS failure
Did HACLS fail ?
NO
YES
Subroutine MAINT (INDEX)
Was failure on an aircraft whose HACLS was checked ?
NO
HSTAT(INDEX) = 2
YES
HSTAT(INDEX) = 3
Is a HACLS presently in maintenance ?
NO
Schedule end of maintenance event
YES
File this aircraft's HACLS in maintenance QUEUE
Was failure on a tentatively assigned alert aircraft ?
NO
YES
Call REPLAC to reassign on Aircraft to alert mission

FIGURE 8B

or equal to 12 hours or launch time greater than 12 hours. Then a Monte Carlo check compares a random number against the reliability function value,

$$R_i = \exp(-k_j \lambda_L t_{ij}), \qquad t_{ij} \geq 0.$$

The t_{ij}'s come from either Table III, if launch time $\leq$ 12 hours or Table IV, if launch time > 12 hours after the start of the simulated day. No updating of HSTAT is done, since this is a pseudo-launch, used only to determine the value of the Bernoulli performance variable. Two counter variables are introduced, one (NALERT) to count the number of alert launch replications, and the other (NOK) to count the number of successful alert launches.

"Successful" is defined in two different ways: (i) The two alert aircraft are launched, and one non-alert aircraft is launched; if the HACLS belonging to all three aircraft are up, the launch is considered successful (refer to figure 7A). A_{est} is then calculated using equation 2a. Since A_{est} is based on the availability of three aircraft it should be approximately equal to A^3 as shown in Chapter VII. (ii) The HACLS status (HSTAT) of each of the squadron's aircraft is determined and a tally of UP HACLS, by aircraft, is kept for the 1601 replications. The resulting number of successes is divided by the number of trials for each aircraft, and then averaged over all the aircraft.

TABLE III

PREVIOUS DAY MISSION ASSIGNMENTS	HACLS MODE INDEX 1	2	3	4	5	6
1 (ALERT 1)	10	0	1.98	0.02	0	0
2 (ALERT 2)	10	0	1.98	0.02	0	0
3 (OPERATIONAL 1)	4	8	0	0	0	0
4 (OPERATIONAL 2)	2	10	0	0	0	0
5 (TRAINING A)	9	3	0	0	0	0
6 (TRAINING B)	9	3	0	0	0	0
7 (TRAINING C)	8	4	0	0	0	0
8 (NON-FLY 1)	12	0	0	0	0	0
9 (NON-FLY 2)	12	0	0	0	0	0

T1 MATRIX

Entries are the number of hours by mission category that HACLS is in the various operating modes.

TABLE IV

PREVIOUS DAY MISSION ASSIGNMENTS	HACLS MODE INDEX 1	2	3	4	5	6
1 (ALERT 1)	12	0	0	0	0	0
2 (ALERT 2)	12	0	0	0	0	0
3 (OPERATIONAL 1)	2	10	0	0	0	0
4 (OPERATIONAL 2)	4	8	0	0	0	0
5 (TRAINING A)	12	0	0	0	0	0
6 (TRAINING B)	12	0	0	0	0	0
7 (TRAINING C)	12	0	0	0	0	0
8 (NON-FLY 1)	12	0	0	0	0	0
9 (NON-FLY 2)	12	0	0	0	0	0

T2 MATRIX

Entries are the number of hours by mission category that HACLS is in the various operating modes.

Figure 8A shows this alternative calculation of availabilities, A_i, $i=1,...,9$, and averaging to get

$$\bar{A} = (1/n) \sum_{i=1}^{n} A_i.$$

In the simulation runs both A_{est} and $\bar{A}$ are calculated. A_{est} may be considered an operational availability in the sense that it represents the launch success probability as seen by an operational commander. $\bar{A}$, on the other hand, represents a logistic availability since it is based upon sampling HACLS status of all aircraft assigned to the squadron, and not just the ones which would be tasked to fly the strike missions. The following asymptotic availability is computed:

$$A = 1 - (\text{Down Time})/(\text{Total Time}). \qquad (4)$$

By probability theory, equation 4 is mathematically equivalent to the standard definition of asymptotic availability (equation 1). However, since the simulation is responsive to maintenance efficiency, α ($\alpha < 1.0$), the resultant value of A should correspond to that calculated using equation 3. This availability calculation is made in order to verify the other availability computations described above.

To parametrically insert HACLS flights into the daily operation scheme, an indicator variable, IHAC3 (not shown in the flow charts), is set to 1 for a day with a HACLS flight

or 0 for a day without a HACLS flight. On days with a HACLS flight the aircraft assigned to mission number 3 files the Harpoon mission. Therefore, R_3 is modified to reflect the more severe HACLS operating environment, as follows:

$$R_3 = \exp(-k_j \lambda_L t_{3j}), \qquad j=1,\ldots,6;$$

where t_{3j} comes from Table V.

TABLE V

PREVIOUS DAY MISSION ASSIGNMENT	HACLS MODE INDEX 1	2	3	4	5	6
3(OPERATIONAL 1)	12	4	1.98	0.02	5.98	0.02

T3 VECTOR

After approximately 1601 replications the run is terminated and the results are printed out. These results include values of availability calculated using equation 2a with the counter variables NOK, NOK1(•), and NALERT; availability from equation 4, and $\overline{A}$. The actual computations of the availabilities take place at points where no HACLS are in a failed state. Therefore, if at 1601 replications there exist any failed HACLS, the simulation run is continued until they have been repaired. Additional runs with parametric variations of α are made and after all are completed, the simulation terminates.

An earlier version of this model is in published form as [10]. Changes incorporated in the present model include a random aircraft/mission rotation scheme having replaced a deterministic one, and the parametric incorporation of missile - carrying training flights in the simulation. Additionally, in order to verify the response of the computer model and to validate some assumptions of the analytical model, parametric runs varying maintenance time (U) distribution type and mean time to repair, μ, are made.

Additions to the simulation in support of the analytical model include: i) More than one method of availability computation is employed and ii) A temporary program change is used to empirically determine the distribution of idle time periods.

In addition to the above support, sensitivity analyses performed using the simulation model are bases for some assumptions of the analytical model. Therefore, the GASP IV model provides a basis for verifying the Markov chain, alternating renewal process model and for validating some of its assumptions. Furthermore, because sampling in the present model occurs at points in simulated time at which no HACLS are failed, a previously programmed "steady-state" check has been eliminated.

D. DISCUSSION

The assignment of two aircraft to alert status is consistent with existing operational procedures at P-3 overseas deployment sites. That is, the alert mission used

in this model exists in reality to provide ready aircraft for ASW, SAR, and surveillance missions, as well as for quick-load weapons-carrying missions, which in the future would include Harpoon. Since these ready-alert aircraft receive complete preflight checks daily, it is a rare event that either of them would be in a HACLS failed status. Since the P-3 operational commander requires fully operational aircraft for the alert role, once a failure in any system is detected during an alert aircraft preflight, either the failure is isolated and the faulty component replaced, or a fully up aircraft is substituted for the faulty one. In this model, the latter worst-case situation is assumed when a HACLS fails.

The criterion for determining the number of successful alert launches for the calculation of availability using equation 2 is considered a workable one since, over the approximately 1601 replications of alert launches, the HSTAT of the third aircraft in the alert strike force would be representative of a typical nonalert aircraft. Thus, it would provide a reasonable sample for availability determination. This contention is borne out when one compares this value of A_{est} with $\overline{A}$.

Concerning the scheduling of the end of HACLS maintenance event, it is felt that, because the weapon replaceable assemblies (WRAs) are intended to be removed and replaced immediately upon detection of a failure, a worst-case analysis model would consider that replacement spares are not

immediately available. Alternatively, one could consider that the failure is not immediately isolated and therefore prompt HACLS repair by WRA replacement is not feasible. Nonetheless, this simulation labels each failed WRA with its respective aircraft identification number and, depending upon the Navy Intermediate Maintenance Activity (IMA) test bench availability, the WRA is placed on the bench for repair or is placed in a maintenance queue. This model assumes that only one IMA test bench is available to service all WRAs. Moreover, although more than one type of WRA exists in reality, WRAs are assumed to be indistinguishable; thus, the worst-case nature of this model is accentuated.

In effect this model and the analytical one give no supply support to the squadron, a seemingly unrealistic state of nature. Nevertheless, since HACLS is not presently installed in operational squadron aircraft, the time to repair probability distribution is unknown. Therefore, because a distribution has to be assumed in any case, the supply lag-time, if any, is aggregated with the maintenance time to repair. By considering maintenance to repair distribution as containing times to replace faulty components, as well as the time awaiting parts not readily available from the supply system, one can see that realism is preserved.

An estimate of the cost of computer facilities used to develop the computer simulation model is $1066. Estimated programmer time involved is 160 hours.

VI. INTERVAL AVAILABILITY

So far, the discussion has concerned itself with point availability, the probability that a randomly selected HACLS is up at a random point in time. The interval availability, on the other hand, is defined as the expected fraction of a given interval of time that a system will be able to operate within its design tolerances [3]. The symbol A* will be used to denote interval availability.

$$A^* = A \exp(-\lambda^{*\prime} t), \tag{5}$$

where

A = point availability determined from the analytical model;

λ^* = the failure rate vector, for HACLS under its critical mission operating environment (eg. strike flight);

and

t = the vector of time in each HACLS mode.

Assumptions needed for equation 5 are: (i) No start-up shock effect when HACLS shifts from one mode of operation to another. (ii) Adverse effects associated with one HACLS operation mode are independent of those existing in a subsequent mode.

For a critical mission of 10 hours, for example, with HACLS in the non-operating flight mode for 1.98 hours, BIT

mode for 0.02 hour, and standby flight mode for one hour, the $\lambda^{*'}$ vector would be (0.1. 6.0, 5.0) λ_L and t' is (1.98, 0.02, 8.0). Given an availability, A, of 0.975, for example,

$$A^* = 0.975 \quad \exp(-\lambda^{*'}t) = 0.933.$$

VII. PRESENTATION OF RESULTS

Given the MDAC-E reliability figures for laboratory failure rate, λ_L; maintenance time to repair, U, assumed to be exponential, with a mean of 24 hours; and the P-3 environmental k-factors, listed in Table I; the analytical model yields an A of 0.998 for the base case of one HACLS flight per week, and maintenance efficiency, α, of 0.8. The simulation, on the other hand, results in an asymptotic availability of 0.997, with the same initial parameter values. Equation 3 provides A = 0.998 to favorably compare with these results. The analytical model gives a π_1 of 0.949, the probability that HACLS is up at the beginning of an operating period. A_{est} and $\overline{A}$ from the simulation equal 0.988.

In order to test sensitivity of the model to changes in reliability, R, the operational scenario may be varied using the T matrix shown in Table VI. Although this variation presumes much greater HACLS utilization in the simulation, it is considered for two reasons: i) to check the responsiveness of the model to an increased number of severe HACLS operating environments, and ii) to determine the effect of daily checks of HACLS on its point availability.

Variations in mean time to repair, μ, yield the results shown in figure 9. The figure indicates that A is relatively

TABLE VI

PREVIOUS DAY MISSION ASSIGNMENTS	HACLS MODE INDEX 1	2	3	4	5	6
1 (ALERT 1)	22	0	1.98	0.02	0	0
2 (ALERT 2)	22	0	1.98	0.02	0	0
3 (OPERATIONAL 1)	13.5	10	0.48	0.02	0	0
4 (OPERATIONAL 2)	13.5	10	0.48	0.02	0	0
5 (TRAINING A)	20.5	3	0.48	0.02	0	0
6 (TRAINING B)	20.5	3	0.48	0.02	0	0
7 (TRAINING C)	19.5	4	0.48	0.02	0	0
8 (NON-FLY 1)	23.5	0	0.48	0.02	0	0
9 (NON-FLY 2)	24	0	0	0	0	0

MODIFIED T MATRIX

Entries are the number of hours by mission category that HACLS is in the various operating modes.

insensitive to maintenance distribution, but is, of course, sensitive to mean time repair.

Maintenance efficiency, α, increased beyond 0.8 does not appreciably increase availability, as shown in figure 10. However, availability at $\alpha = 0.5$ is significantly below that of $\alpha = 0.8$, the base efficiency.

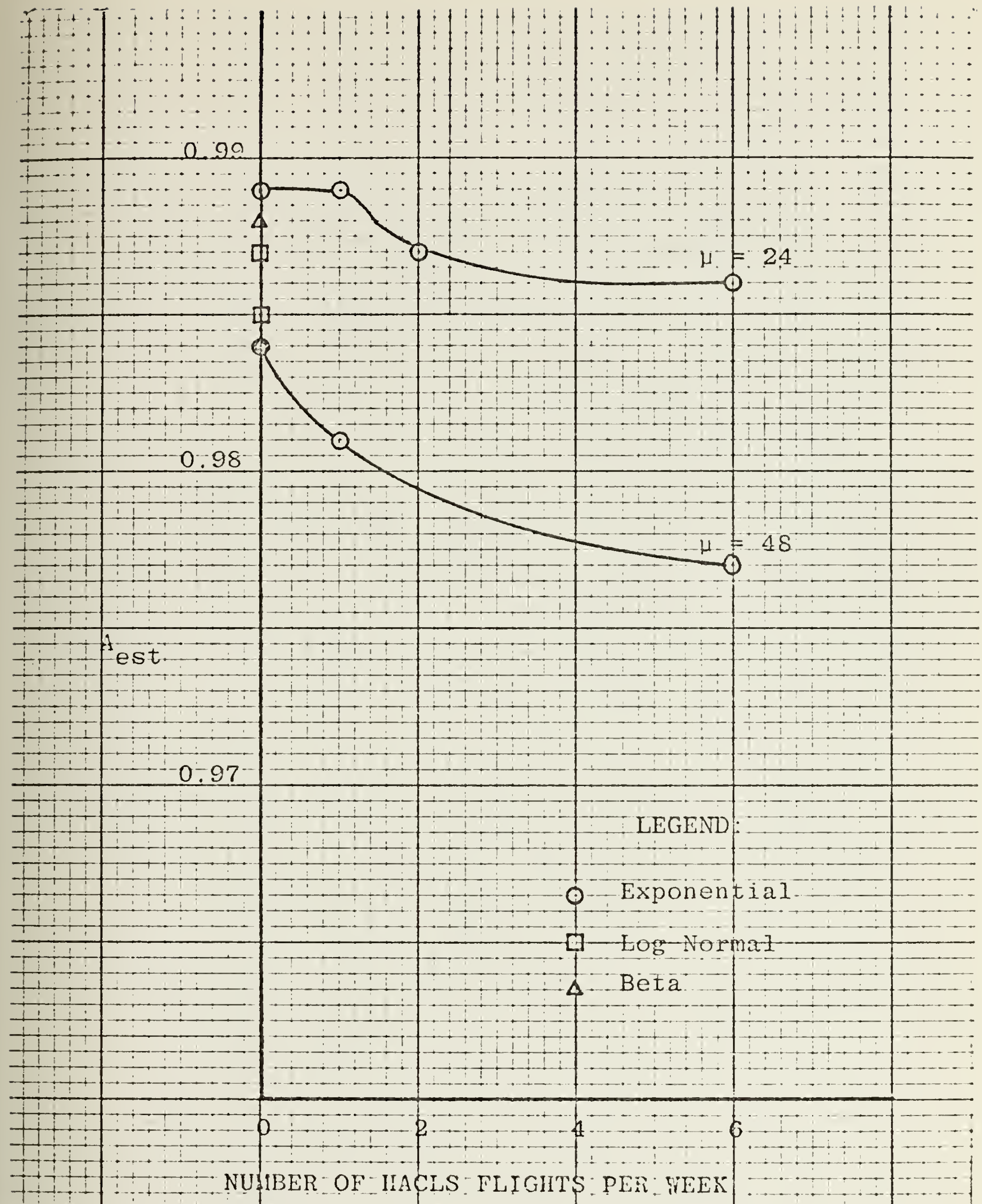
The analytical model is exercised with variations in the number of HACLS flights per week, and the combined results are shown in Table VII. Results for A-6, P-3, and S-3A squadrons are summarized in Table VIII. It is clear from the above discussion that the semi-Markov model appears to be robust for non-exponential (non-Markovian) cases examined empirically.

On the basis of available numbers of aircraft by type, the total number of failures per year are [11]

	No Flights	1 Flight per Mo.
P-3 (324 A/C)	118	290
S-3A (180 A/C)	55	115
A-6 (384 A/C)	200	246
Total (888 A/C)	373	651

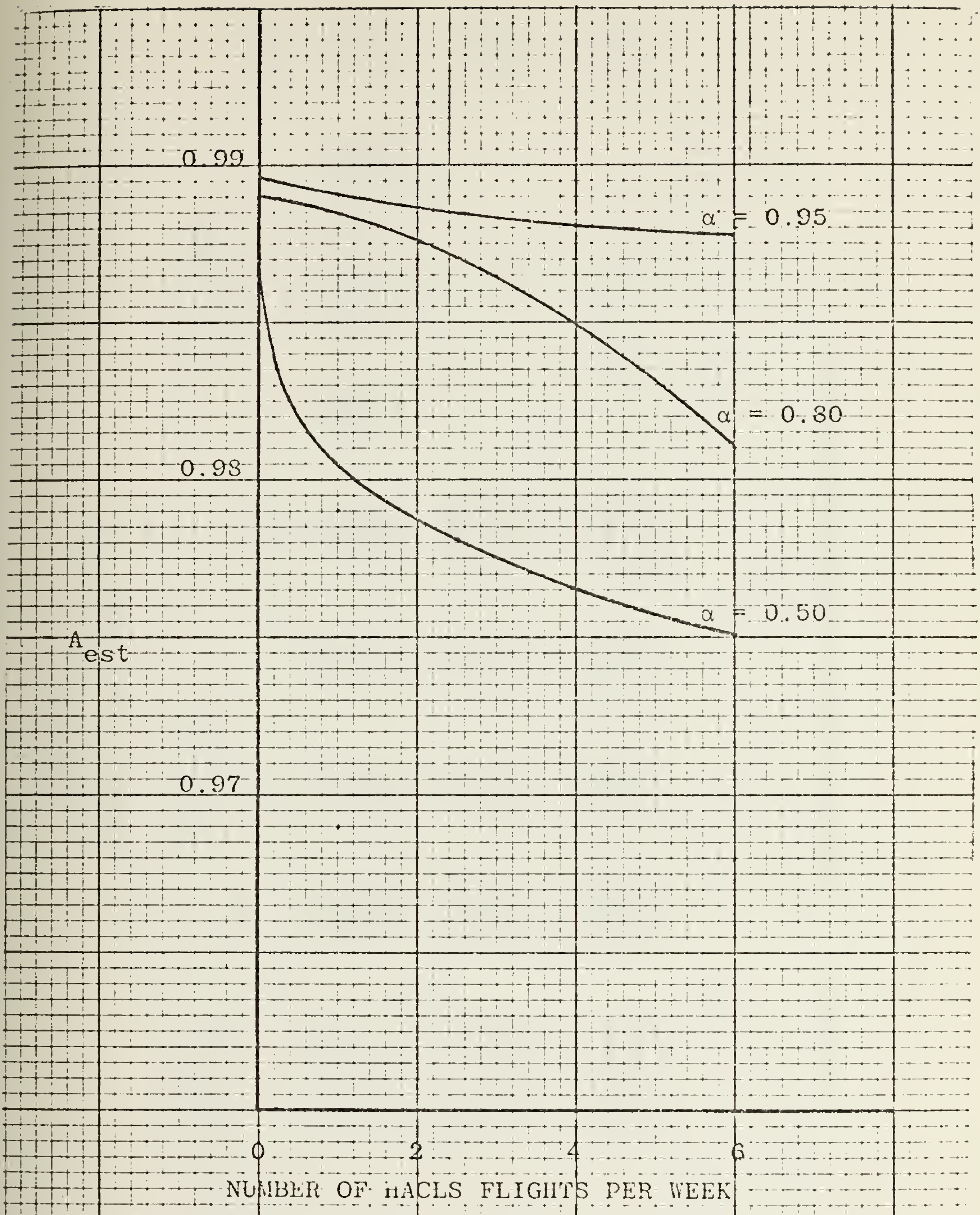
Assuming the below listed percentages of failures for each WRA [11], the breakdown of failures by components is:

		No. Flights	1 Flight per Mo.
MCP	6.8%	25	44
HDP	47.7%	178	311
AAIU	27.3%	102	178
HLU	11.8%	44	77
Others	6.4%	24	41
	100.0%	373	651



A_{est} versus Number of HACLS flights per week with parametric variations of μ and distribution of U.

FIGURE 9



A_{est} versus Number of HACLS flights per week with parametric Variations of α.

FIGURE 10

TABLE VII

1. T Matrix from Table II.

No. of HACLS	CASE	SIMULATION				ANALYTICAL			
Flights per Week	(TTF-TTR)	A_{est}	$A_{est}^{1/3}$	$\overline{A}$	A (eq.4)	π_1	$E[O_i+I_i]$	A	A (eq.3)
0	exp-exp	0.989	0.996	0.991	0.998	0.950	92.0	0.998	0.999
1	exp-exp	0.989	0.996	0.992	0.998	0.946	92.3	0.998	0.998
2	exp-exp	0.978	0.993	0.984	0.995	0.931	87.6	0.997	0.998
6	exp-exp	0.986	0.995	0.988	0.993	0.909	78.7	0.995	0.996
0	exp-logn	0.990	0.996	0.992	0.999				0.999
1	exp-logn	0.991	0.997	0.992	0.998				0.998
2	exp-logn	0.991	0.997	0.990	0.998				0.998
6	exp-logn	0.980	0.993	0.985	0.993				0.996
0	exp-beta	0.989	0.996	0.990	0.998				0.999
1	exp-beta	0.990	0.996	0.992	0.998				0.998
2	exp-beta	0.993	0.998	0.993	0.998				0.998
6	exp-beta	0.983	0.994	0.988	0.995				0.996

Comparison of Analytical Model and Simulation Results

TABLE VII (continued)

2. T Matrix from Table VI.

No. of HACLS	CASE	SIMULATION				
FLIGHTS PER WEEK	(TTF-TTR)	A_{est}	$A_{est}^{1/3}$	$\overline{A}$	A (eq.4)	A (eq.3)
0	exp-exp	0.987	0.996	0.990	0.997	0.999
1	exp-exp	0.988	0.996	0.988	0.997	0.998
2	exp-exp	0.982	0.994	0.982	0.996	0.998
6	exp-exp	0.968	0.989	0.976	0.992	0.996
0	exp-logn	0.990	0.997	0.989	0.998	0.999
1	exp-logn	0.988	0.996	0.986	0.997	0.998
2	exp-logn	0.992	0.997	0.991	0.997	0.998
6	exp-logn	0.980	0.993	0.985	0.994	0.996
0	exp-beta	0.987	0.996	0.983	0.998	0.999
1	exp-beta	0.987	0.996	0.985	0.997	0.998
2	exp-beta	0.984	0.995	0.989	0.997	0.998
6	exp-beta	0.988	0.996	0.990	0.995	0.996

Comparison of Analytical Model and Simulation Results

TABLE VIII

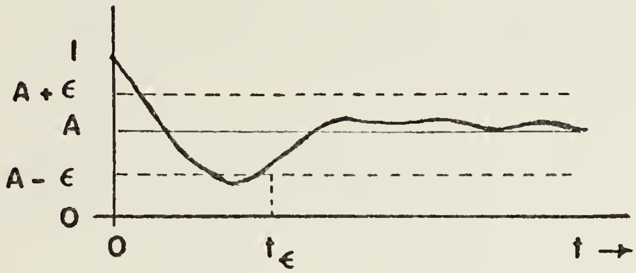
TYPE OF A/C	NO. OF HACLS FLIGHTS PER MONTH	π_1	A	E[Failures/A/C/Yr]	k_{avg}
P-3	0	0.950	0.999	0.36	0.03753
S-3A	0	0.950	0.999	0.30	0.03133
A-6	0	0.948	0.998	0.52	0.05370
P-3	1	0.948	0.998	0.89	0.09261
S-3A	1	0.944	0.997	0.63	0.06515
A-6	1	0.947	0.998	0.64	0.06689

Combined Parametric Results for all Types of Aircraft Determined from the Analytical Model.

APPENDIX A

PROOF OF THE THEOREM.

Since A exists, then for any $\varepsilon > 0$, there exists a t_ε such that $A - \varepsilon \leq A(t) \leq A + \varepsilon$ when $t > t_\varepsilon$.



Also, for $t > t_\varepsilon$,

$$\int_0^t A(s)ds = \int_0^{t_\varepsilon} A(s)ds + \int_{t_\varepsilon}^t A(s)ds,$$

so that

$$\int_0^{t_\varepsilon} A(s)ds + \int_{t_\varepsilon}^t (A-\varepsilon)ds \leq \int_0^t A(s)ds \leq \int_0^{t_\varepsilon} A(s)ds + \int_{t_\varepsilon}^t (A+\varepsilon)ds.$$

Now dividing by t in the inequality above, and letting $t \to \infty$

$$\lim_{t\to\infty} \frac{1}{t} \int_0^{t_\varepsilon} A(s)ds = 0,$$

$$\lim_{t\to\infty} \frac{1}{t} \int_{t_\varepsilon}^{t} (A \pm \varepsilon)ds = \lim_{t\to\infty} (A \pm \varepsilon)\frac{(t-t_\varepsilon)}{t} = A \pm \varepsilon ,$$

so that

$$A-\varepsilon \leq \lim_{t\to\infty} \frac{1}{t} \int_0^t A(s)ds \leq A + \varepsilon$$

Since ε is arbitrary, let $\varepsilon \to 0$. Then,

$$A \leq \lim_{t\to\infty} \frac{1}{t} \int_0^t A(s)ds \leq A,$$

i.e.

$$\lim_{t\to\infty} \frac{1}{t} \int_0^t A(s)ds = A. \quad \text{Then}$$

recall that

$$E\left[\frac{T_{up}(t)}{t}\right] = \frac{1}{t} \int_0^t A(s)ds, \text{ so that}$$

$$\lim_{t\to\infty} E\left[\frac{T_{up}(t)}{t}\right] = A.$$

APPENDIX B

P-3
NO. HACLS FLIGHTS

78840 is the number of aircraft hours in one year of P-3 squadron operations (9 a/c x 8760 hours/year).

	t	k	kt
78840 - 11680	= 67160	0.01	671.6
365 x 30	= 10950	0.1	1095
365 x (1.98)(2)	= 14454	1	1445.4
365 x (0.02)(2)	= 14.6	6	87.6
	0	5	0
	0	6	0

$$\frac{\Sigma kt}{78840} = k_{avg} = 0.04176$$

1 FLIGHT PER WEEK

	t	k	kt
78840 - 12514	= 66326	0.01	663.26
52x24+313x30	= 10638	0.1	1063.8
52x3(1.98)+313(2)(1.98)	= 1548.4	1	1548.4
52x3(0.02)+313(2)(0.02)	= 15.64	6	93.84
52 x 5.98 + 0	= 310.96	5	1554.8
52 x 0.02	= 1.04	6	6.24

$$k_{avg} = 0.0625$$

2 FLIGHTS PER WEEK		t	k	kt
78840 - 12618	=	66222	.01	662.22
104x24+261x30	=	10326	.1	1032.6
104x3(1.98)+261(2)(1.98)	=	1651.32	1	1651.32
104x3(0.02)+261(2)(.02)	=	16.68	6	100.08
104 x 5.98	=	621.92	5	3109.6
104 x 0.02	=	2.08	6	12.48

$$k_{avg} = 0.0832$$

6 FLIGHTS PER WEEK				
78840 - 12246	=	66594	.01	665.94
52x30+313x24	=	9072	.1	907.2
52x2(1.98)+313x3(1.98)	=	2065.1	1	2065.1
52x2(0.02)+313x3(0.02)	=	20.86	6	125.16
52 x 0 + 313 x (5.98)	=	1081.7	5	5408.7
313 x (0.02	=	6.26	6	37.56

$$k_{avg} = 0.1168$$

The figures below are provided from ref. 12.

	LOW UTILIZATION			NORMAL UTILIZATION			HIGH UTILIZATION			WORST-CASE UTILIZATION		
P-3 HACLS												
GROUND OFF	.01	619.8	6.20	.01	618.8	6.19	.01	609.8	6.10	.01	609.6	6.10
FLIGHT OFF	.1	100	10.00	.1	92.0	9.20	.1	50	5.00	.1	-	
GROUND STDBY	1.0	10	10.00	1.0	11	11.00	1.0	20	20.00	1.0	20	20.00
FLIGHT STDBY	5.0	-		5.0	7.98	39.90	5.0	49.9	249.50	5.0	99.8	499.20
GROUND OPER	6.0	.2	1.20	6.0	.2	1.20	6.0	.2	1.20	6.0	.2	1.20
FLIGHT OPER	6.0	-		6.0	.02	.12	6.0	.1	.60	.2	.2	1.20
		730.0	27.40		730.00	62.61		730.0	282.40		730.0	527.50
	$k_{avg} = .03753$			$k_{avg} = .09261$			$k_{avg} = .38685$			$k_{avg} = .72260$		
S-3 HACLS												
GROUND OFF	.01	664.8	6.65	.01	664.3	6.65	.01	659.8	6.60	.01	659.8	6.10
FLIGHT OFF	.167	60	10.02	.167	55	9.19	.167	30.0	5.01	.167	-	
GROUND STDBY	1.0	5	5.00	1.0	5.5	5.50	1.0	10.0	10.00	1.0	10.0	10.00
FLIGHT STDBY	5.0	-		5.0	4.98	24.90	5.0	29.9	149.50	5.0	59.8	299.00
GROUND OPER	6.0	.2	1.20	6.0	.2	1.20	6.0	.2	1.20	6.0	.2	1.20
FLIGHT OPER	6.0	-		6.0	.02	.12	6.0	.1	.60	6.0	.2	1.20
		730.0	22.87		730.00	47.56		730.0	172.91		730.0	318.00
	$k_{avg} = .03133$			$k_{avg} = .06515$			$k_{avg} = .23686$			$k_{avg} = .43562$		

	LOW UTILIZATION			NORMAL UTILIZATION			HIGH UTILIZATION			WORST-CASE UTILIZATION		
ATTACK HACLS												
GROUND OFF	.01	679.6	6.80	.01	679.1	6.79	.01	669.6	6.70	.01	669.2	6.69
FLIGHT OFF	.5	40.0	20.00	.5	38	19.00	.5	20.0	10.00	.5	-	
GROUND STDBY	1.0	10.0	10.0	1.0	10.5	10.50	1.0	20.0	20.00	1.0	20.0	20.00
FLIGHT STDBY	5.0	-		5.0	1.98	9.90	5.0	19.8	99.00	5.0	37.6	198.00
GROUND OPER	6.0	.4	2.40	6.0	.4	2.40	6.0	.4	2.40	6.0	.4	2.40
FLIGHT OPER	6.0	-		6.0	.2	.24	6.0	.2	1.20	6.0	.4	2.40
		730.0	39.20		730.00	48.83		730.0	119.30		730.0	229.49
	k_{avg} = .05370			k_{avg} = .06681			k_{avg} = .19082			k_{avg} = .31437		

APPENDIX C

The entries in P_A are calculated as follows.

Let $\mathcal{O}(s)$ be the Laplace transform of the density function for O_i.

Then,

$$Pr[Y_i=1|X_i=1,O_i=u]= \frac{\lambda}{\lambda+1/\mu} + \frac{1/\mu}{\lambda+1/\mu}\, e^{-(\lambda+1/\mu)u},$$

and

$$Pr[Y_i=1|X_i=1]= \frac{\lambda}{\lambda+1/\mu} + \frac{1/\mu}{\lambda+1/\mu}\, \mathcal{O}(\lambda+1/\mu),$$

since, letting $O(t) = Pr[O_i \leq t]$,

$$P[Y_i=1|X_i=1]= \int_0^{\infty} (g+he^{-\beta U})dO(u)$$

$$= gO(u)\Big|_0^{\infty} +h \int_o^{\infty} e^{-\beta U}\, dO(u)$$

$$= g + h\mathcal{O}(\beta),$$

where $g = \frac{\lambda}{\lambda+1/\mu}$, $h = \frac{1/\mu}{\lambda+1/\mu}$, and $\beta = \lambda +1/\mu$.

Likewise,

$$Pr[Y_i=2|X_i=1] = 0,$$

$$Pr[Y_i=3|X_i=1] = \frac{1/\mu}{\lambda+1/\mu}\, [1+\mathcal{O}(\lambda+1/\mu)],$$

$$Pr[Y_i=1|X_i=2] = \frac{\lambda}{\lambda+1/\mu}\, [1-\mathcal{O}(\lambda+1/\mu)],$$

$$\Pr[Y_i=2|X_i=2] = 0,$$

$$\Pr[Y_i=3|X_i=2] = \frac{1/\mu}{\lambda+1/\mu} + \frac{\lambda}{\lambda+1/\mu}\, \mathcal{O}(\lambda+1/\mu),$$

$$\Pr[Y_i=1|X_i=3] = \frac{\lambda}{\lambda+1/\mu}\, [1-\mathcal{O}(\lambda+1/\mu)],$$

$$\Pr[Y_i=2|X_i=3] = 0, \text{ and}$$

$$\Pr[Y_i=3|X_i=3] = \frac{1/\mu}{\lambda+1/\mu} + \frac{\lambda}{\lambda+1/\mu}\, \mathcal{O}(\lambda+1/\mu).$$

The entries in P_B are calculated in a similar way.

Let $\mathcal{I}(s)$ be the Laplace transform of the density function for I_i.

Then,

$$\Pr[X_{i+1}=1|Y_i=1] = \mathcal{I}(\lambda),$$

$$\Pr[X_{i+1}=2|Y_i=1] = 1-\mathcal{I}(\lambda),$$

$$\Pr[X_{i+1}=3|Y_i=1] = 0,$$

$$\Pr[X_{i+1}=1|Y_i=3] = \frac{\alpha/\mu}{1/\mu-\lambda}\, [\mathcal{I}(\lambda)-\mathcal{I}(1/\mu)],$$

$$\Pr[X_{i+1}=2|Y_i=3] = \alpha\left[1+ \frac{\lambda\mathcal{I}(1/\mu)}{1/\mu-\lambda} - \frac{1/\mu\mathcal{I}(\lambda)}{1/\mu-\lambda}\right]$$

$$= 1-\mathcal{I}(1/\mu)- \frac{\alpha/\mu}{1/\mu-\lambda}[\mathcal{I}(\lambda)-\mathcal{I}(1/\mu)],$$

and

$$\Pr[X_{i+1}=3|Y_i=3] = \mathcal{I}(1/\mu).$$

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